Analyzing the Effectiveness of the Solow Model in the 21st Century

TROY R. SHEN

Journal of Secondary and Undergraduate Research
Published on November 1st, 2025

It has been nearly seventy years since the creation of the Solow-Swan model, which has been the basis of measuring long term economic growth in the economic field. We hoped to test the accuracy of the Solow model in modern times and to see what results we could get to further gain insight on growth in developing and developed countries. This paper evaluates the effectiveness of the Solow model by creating a value for elasticity of output with respect to capital from the US Solow model by using sensitivity analysis, and then creating Solow models for Germany, India and Japan using formulas created to estimate capital, labor, and technological growth. We found that under the right conditions, the Solow model can be effectively applied to predict economic growth yet fails to address many problems like corruption or government policies.

1 Introduction

Since the dawn of economic theory, economists have created and debated many methods of predicting economic growth. One such model, created by Robert Solow and Trevor Swan in 1956, would become the basis of modern neoclassical growth theory. By using four factors (capital, elasticity of output with respect to capital, labor, and technology), the Solow-Swan model predicts the total production of a country [1].

Capital has been a reliable basis for most models of growth, as it is a necessary factor in production. Capital can be split into public capital, which consists of infrastructure, electricity, and other public utilities, and private capital, which for the most part is private machines, databanks, etc. Labor is also vital in measuring growth. A growing population means more people to operate and create capital, stimulating growth. The efficiency of the workforce is also an important factor to note. Technological advances like the internet have drastically boosted our workforce's knowledge base and efficiency, and further technological developments like artificial intelligence can further boost our efficiency. The Solow model aims to combine these simple factors to predict future growth.

However, there are many limitations to this model. For example, the model assumes there is no specific government (which could implement policies) and no international trade [2]. The model also fails to address world events which may change population growth, capital growth, and technological growth. The model is also rather vague in the ways of measuring capital and technological growth. Technology has drastically changed since then, and a series of economic recessions have changed the way we run our economy. The Solow model was an early attempt to measure long-run economic growth, so will this model still be a viable way of predicting economic growth?

In this paper, we test the validity of this model nearly seventy years after being first published. We first apply the Solow model to the US to calculate an elasticity of output with respect to capital, then compare the Solow models of Germany, India, and Japan with their respective GDPs to gauge the accuracy of the Solow model.

2 Methodology

The Solow model uses four main factors: capital (K), elasticity of output with respect to capital (α), labor (L), and technology (A). The Solow model equation is:

$$Y(t) = K(t)^a [A(t)L(t)]^{1-a} \quad (1)$$

Other than the assumptions made in the Solow model mentioned earlier, we make a few more key assumptions. The first is that when calculating formulas which best represent Capital, Labor, and Technology, we change the type

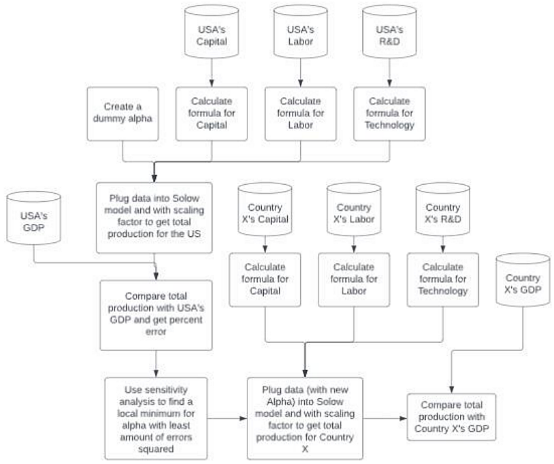

Figure 1: Flowchart for Calculating a Base Elasticity of Output with Respect to Capital (α) from the US and Future Comparisons with Other Countries.

of function (linear, exponential, polynomial, etc.) based on the plotted graph of actual data. We tried at first to model Technology and Labor under the constraints of the Solow model (labor and technology are exponential) [5], but our data did not work out very well. We also defaulted to a linear function for Capital's formula, as Capital depreciates over time. In addition, we also use a scaling constant to compare GDP and the total production calculated by the Solow model. Dimensional analysis does not seem to work here with international dollars by the IMF (will explain later) and people in comparison with GDP. We believe that using a scaling factor will solve this problem, as we are measuring the changes between our calculated Solow model production and GDP.

In our calculations, for capital we use IMF calculated Private Capital Stocks in international dollars and for technology we use Country X's research and development funding (R&D) in international dollars. IMF International dollars are a combination of multiple national currencies to better compare data between countries. The data we use in GDP is also in international dollars. We took data from 1995 to 2015 on the United States, Germany, India, and Japan to create formulas which predicted each factor after 1995.

Figures 2-4 demonstrate our method mentioned earlier of calculating formulas which predict factors for the US. For the most part (other than the cyclical nature of the graph), data collected about the United States created sensible formulas. However, this was not the case for all four countries.

Figure 5 shows a discrepancy with calculating a formula for German capital. Around 2003, German private capital stocks begin to decrease. Because Germany has had a decrease in investment (both public and private) since 2003 [6], it would not be accurate to limit our formula to once single function; instead, we created a piecewise function with different 'regime' changes to better predict German capital stocks. Since the 2000's, Germany has displayed

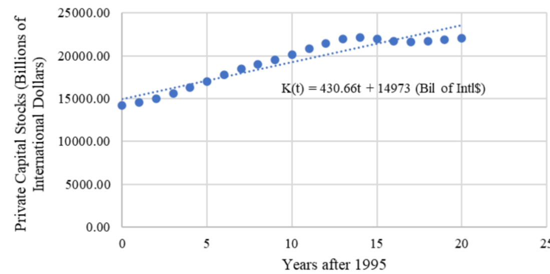

Figure 2: Graph used to Calculate Formula Predicting US Capital after 1995.

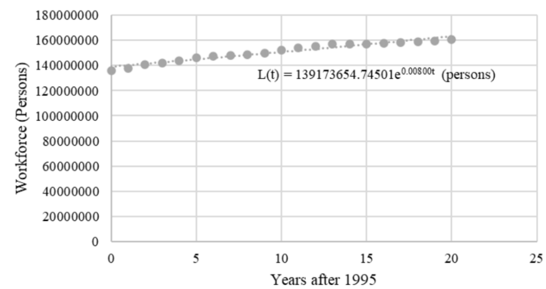

Figure 3: Graph used to Calculate Formula Predicting US Labor Force after 1995.

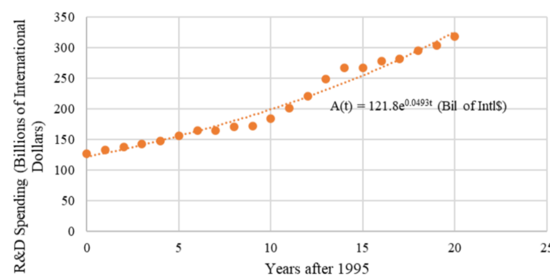

Figure 4: Graph used to Calculate Formula Predicting US Technological Development after 1995.

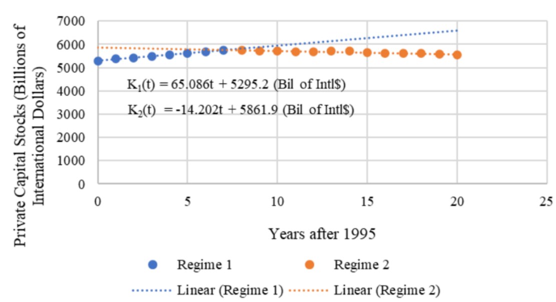

Figure 5: Graph used to Calculate Formula Predicting German Capital after 1995.

low and negative public fixed net capital formation ratios, meaning slowed economic growth [6]– which pertains to the Solow model.

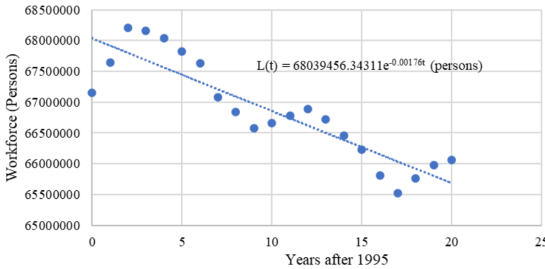

Figure 6: Graph used to Calculate Formula Predicting Japanese Labor Force after 1995.

Figure 6 also shows two additional problems: the first being a decreasing population, the second being unable to capture the cyclical nature of this data. Japan, similarly, to China, is undergoing a declining population – which is not so great for economic growth. Declining and aging populations push a lot of the burden onto the already decreasing younger generations in paying for social welfare. Although the first problem is not an immediate one for the Solow model, the second casts some uncertainty of the accuracy of this formula. We are dictated by the Solow model to use an exponential function in capturing labor and technology.

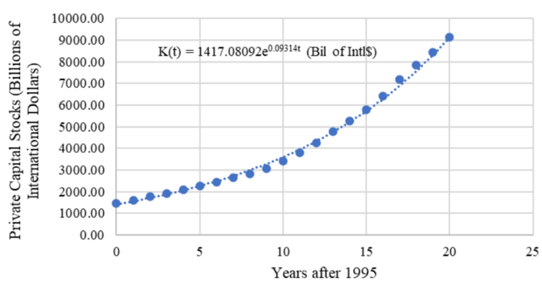

Figure 7: Graph used to Calculate Formula Predicting Indian Capital after 1995.

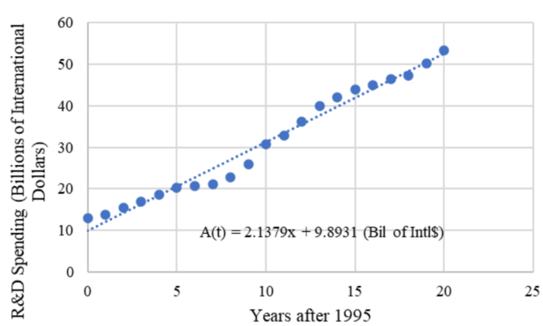

Figure 8: Graph used to Calculate Formula Predicting Indian Technology after 1995.

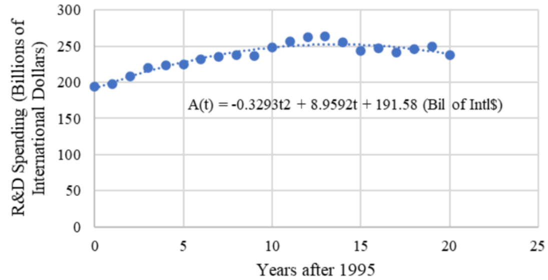

Figure 9: Graph used to Calculate Formula Predicting Japanese Technology after 1995.

Figures 7-9 show the rest of the discrepancies for creating formulas for capital, labor, and technology. In Figure 9, Japan sees a stagnation in technological investment. This is likely in part because of weak investments from private companies as they have no incentive to spend [4]. For the rest of the graphs, we used linear functions for capital and exponential functions for labor and technology.

3 Data

Data on individual countries' private capital stocks and GDP were found in the International Monetary Fund database.

Data on individual countries' labor force was found in the World Bank database.

Data on individual countries' research and development expenditures was found in the World Bank database. This data with the IMF data to calculate R&D spending in international dollars.

4 Results

For the US, we have calculated using the Solow Model that their elasticity of output with respect to capital is around 0.526 by using sensitivity analysis. Sensitivity analysis is done by plugging in numbers for alpha to minimize the sum of errors squared (to summarize it is pretty much guess and check). To verify our data, we graph US GDP alongside Y(t) squared in figure 7.

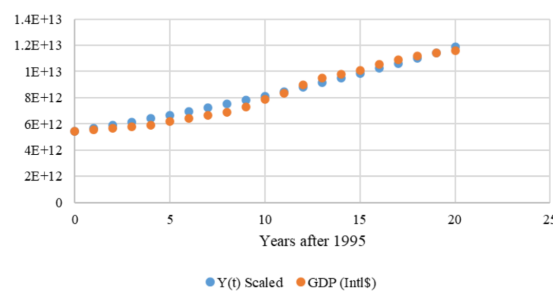

Figure 10: Graph Calculating US Total Production using Solow Model alongside US GDP in International Dollars.

Y(t)	Y(t) Scaled	GDP (Intl$)	Error	$Error^2$	% Error
1.11E+07	5.43E+12	5.43E+12	0.00E+00	0.00E+00	0.0%
1.16E+07	5.66E+12	5.53E+12	1.31E+11	1.73E+22	2.4%
1.21E+07	5.91E+12	5.68E+12	2.29E+11	5.22E+22	4.0%
1.26E+07	6.15E+12	5.79E+12	3.62E+11	1.31E+23	6.3%
1.31E+07	6.41E+12	5.93E+12	4.83E+11	2.33E+23	8.1%
1.36E+07	6.68E+12	6.21E+12	4.62E+11	2.13E+23	7.4%
1.42E+07	6.95E+12	6.43E+12	5.17E+11	2.67E+23	8.0%
1.48E+07	7.23E+12	6.64E+12	5.92E+11	3.50E+23	8.9%
1.53E+07	7.53E+12	6.87E+12	6.78E+11	3.95E+23	9.1%
1.60E+07	7.83E+12	7.30E+12	5.32E+11	2.83E+23	7.3%
1.66E+07	8.14E+12	7.85E+12	2.84E+11	8.09E+22	3.6%
1.73E+07	8.46E+12	8.37E+12	9.26E+10	8.57E+21	1.1%
1.80E+07	8.79E+12	8.97E+12	-1.78E+11	3.15E+22	-2.0%
1.87E+07	9.14E+12	9.48E+12	-3.46E+11	1.19E+23	-3.6%
1.94E+07	9.49E+12	9.79E+12	-2.98E+11	8.88E+22	-3.0%
2.01E+07	9.87E+12	1.01E+13	-2.17E+11	4.72E+22	-2.2%
2.09E+07	1.02E+13	1.05E+13	-2.89E+11	8.34E+22	-2.7%
2.17E+07	1.06E+13	1.09E+13	-2.85E+11	8.12E+22	-2.7%
2.25E+07	1.10E+13	1.12E+13	-1.37E+11	1.89E+22	-1.2%
2.34E+07	1.14E+13	1.14E+13	6.71E+10	4.50E+19	0.1%
2.42E+07	1.19E+13	1.16E+13	3.02E+11	9.10E+22	2.6%
Alpha		0.526	**Scaling Factor**		4.90E+05
Sum Errors Squared		2.59E+24	**Average Error**		2.5%

Table 1: Calculated US Elasticity of Output with Respect to Capital (α) using Sensitivity Analysis.

Our calculated US total production and GDP in international dollars align well with each other. With our new alpha of 0.526, we then plugged in this alpha with capital, labor and technology for Germany. Here were our results:

Y(t)	Y(t) Scaled	GDP (Intl$)	Error	$Error^2$	% Error
7.74E+07	2.78E+12	2.78E+12	0.00E+00	0.00E+00	0.0%
7.96E+07	2.86E+12	2.81E+12	5.40E+10	2.92E+21	1.9%
8.18E+07	2.94E+12	2.86E+12	8.02E+10	6.44E+21	2.8%
8.41E+07	3.02E+12	2.91E+12	1.11E+11	1.23E+22	3.8%
8.63E+07	3.11E+12	2.97E+12	1.40E+11	1.97E+22	4.7%
8.87E+07	3.19E+12	3.06E+12	1.37E+11	1.71E+22	4.5%
9.12E+07	3.28E+12	3.12E+12	1.62E+11	2.61E+22	5.2%
9.36E+07	3.37E+12	3.19E+12	1.68E+11	2.64E+22	5.3%
9.50E+07	3.42E+12	3.09E+12	3.23E+11	1.04E+23	10.4%
9.63E+07	3.46E+12	3.11E+12	3.45E+11	1.19E+23	11.0%
9.74E+07	3.51E+12	3.14E+12	3.14E+11	9.87E+22	9.9%
9.88E+07	3.55E+12	3.27E+12	2.87E+11	8.21E+22	8.8%
1.00E+08	3.59E+12	3.27E+12	2.42E+11	5.87E+22	7.4%
1.01E+08	3.40E+12	3.40E+12	2.48E+11	6.13E+22	7.5%
1.03E+08	3.62E+12	3.21E+12	4.10E+11	1.68E+23	12.8%
1.04E+08	3.69E+12	3.26E+12	4.27E+11	1.82E+23	13.1%
1.05E+08	3.79E+12	3.45E+12	3.43E+11	1.05E+23	9.9%
1.07E+08	3.87E+12	3.47E+12	4.00E+11	1.60E+23	11.5%
1.08E+08	3.89E+12	3.48E+12	3.78E+11	1.43E+23	10.9%
1.09E+08	3.94E+12	3.56E+12	3.69E+11	1.37E+23	10.2%
Alpha		0.526	**Scaling Factor**		1.72E+24
Sum Errors Squared		1.72E+24	**Average Error**		7.8%

Table 2: Calculated German Elasticity of Output with Respect to Capital (with α=0.526) using Sensitivity Analysis alongside German GDP.

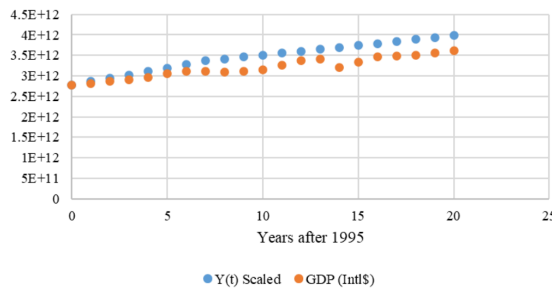

Figure 11: Graph Calculating German Total Production (with α=0.526) using Solow Model alongside German GDP in International Dollars.

For the most part, our data seems to work well here. There is a noticeable gap after 2002, but that aligns with the lack of investment that occurred around 2003. But otherwise, Germany aligns quite well with the predicted Solow Model graph - no problems here. However, we can't say the same for India and Japan.

Y(t)	Y(t) Scaled	GDP (Intl$)	Error	$Error^2$	% Error
2.48E+07	2.02E+12	2.02E+12	0.00E+00	0.00E+00	0.0%
3.02E+07	2.45E+12	2.16E+12	2.83E+11	8.03E+22	13.1%
3.60E+07	2.93E+12	2.16E+12	6.67E+11	4.45E+23	29.5%
4.26E+07	3.47E+12	2.39E+12	1.06E+12	1.14E+24	44.7%
5.00E+07	4.07E+12	2.56E+12	1.49E+12	2.25E+24	58.4%
5.83E+07	4.74E+12	2.67E+12	2.07E+12	4.28E+24	77.4%
6.77E+07	5.50E+12	2.81E+12	2.69E+12	7.24E+24	95.7%
7.61E+07	6.18E+12	2.92E+12	3.26E+12	1.18E+25	112.1%
8.99E+07	7.31E+12	3.16E+12	4.16E+12	1.73E+25	131.4%
1.03E+08	8.40E+12	3.42E+12	4.97E+12	2.47E+25	145.3%
1.18E+08	9.64E+12	3.43E+12	5.87E+12	3.45E+25	165.9%
1.35E+08	1.09E+13	4.09E+12	6.89E+12	4.75E+25	168.9%
1.49E+08	1.19E+13	4.63E+12	7.58E+12	5.75E+25	163.6%
1.75E+08	1.40E+13	5.42E+12	8.61E+12	7.41E+25	158.9%
1.91E+08	1.62E+13	5.05E+12	1.12E+13	1.24E+26	221.3%
2.26E+08	1.83E+13	5.52E+12	1.27E+13	1.63E+26	230.4%
2.56E+08	2.08E+13	5.95E+12	1.49E+13	2.21E+26	250.1%
2.89E+08	2.35E+13	6.41E+12	1.71E+13	2.93E+26	266.0%
3.28E+08	2.67E+13	6.76E+12	2.00E+13	4.00E+26	296.0%
3.70E+08	3.00E+13	7.16E+12	2.29E+13	5.27E+26	320.0%
4.18E+08	3.40E+13	7.69E+12	2.63E+13	6.92E+26	342.0%
Alpha		0.526	**Scaling Factor**		8.13E+04
Sum Errors Squared		2.73E+27	**Average Error**		160.1%

Table 3: Calculated Indian Elasticity of Output with Respect to Capital (with α=0.526) using Sensitivity Analysis alongside Indian GDP.

The immediate problem we notice is that the percentage error is absurdly high. It also seems to be increasing as each year passes. To better understand what is going on, we must look at this problem graphically.

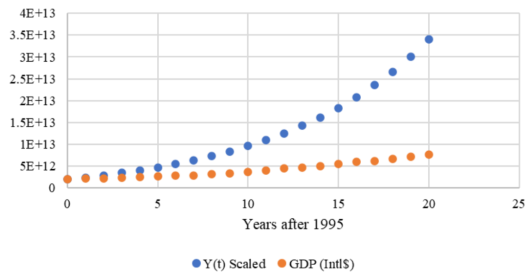

Figure 12: Graph Calculated Indian Total Production (with α=0.526) using Solow Model alongside Indian GDP in International Dollars.

This graph shows us the root of the problem: the Solow Model has drastically overpredicted Indian growth. This problem can be further shown when we set alpha to one instead of 0.526.

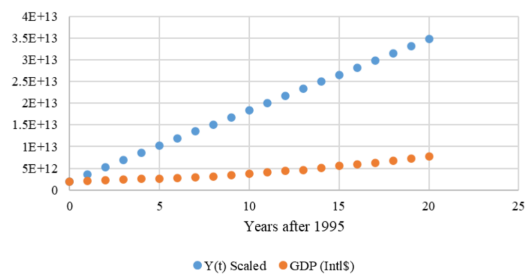

Figure 13: Graph Calculated Indian Total Production (with α=1) using Solow Model alongside Indian GDP in International Dollars.

When alpha equals 1, an increase in capital is equal to an increase in output, leaving no more room for any more growth, thus resulting in a straight increase in GDP. This demonstrates that the Solow model has some faults, as it fails to account that actual growth may be far less than predicted, thus leading to a failure in applying Indian data to the Solow Model. This gap may reflect India's myriad of corruption and institutional problems. Notable problems include a massive rural economy, high unemployment rates, and labor laws, incentivizing employers to hire less if not any workers [3]. What about Japan's situation? Let us now look at Japanese data below:

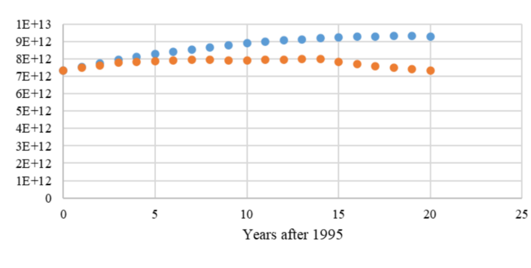

Figure 14: Graph Calculated Japanese Total Production (with α=0.526) using Solow Model alongside Japanese GDP in International Dollars.

Y(t)	Y(t) Scaled	GDP (Intl$)	Error	Error^2	% Error
1.30E+07	7.35E+12	7.35E+12	0.00E+00	0.00E+00	0.0%
1.34E+07	7.56E+12	7.49E+12	7.01E+10	4.92E+21	0.9%
1.37E+07	7.75E+12	7.63E+12	1.18E+11	1.40E+22	1.5%
1.40E+07	7.93E+12	7.78E+12	1.54E+11	2.36E+22	2.0%
1.44E+07	8.10E+12	7.85E+12	2.58E+11	6.64E+22	3.3%
1.46E+07	8.26E+12	7.88E+12	3.85E+11	1.49E+23	4.9%
1.49E+07	8.41E+12	7.99E+12	4.81E+11	2.31E+23	6.1%
1.52E+07	8.57E+12	7.93E+12	6.14E+11	3.77E+23	7.7%
1.54E+07	8.67E+12	7.93E+12	7.39E+11	5.46E+23	9.3%
1.56E+07	8.79E+12	7.93E+12	8.56E+11	7.33E+23	10.9%
1.57E+07	8.91E+12	7.94E+12	9.73E+11	9.48E+23	12.2%
1.60E+07	9.08E+12	7.94E+12	1.14E+12	1.30E+24	14.3%
1.62E+07	9.13E+12	7.99E+12	1.13E+12	1.29E+24	14.2%
1.63E+07	9.19E+12	7.98E+12	1.21E+12	1.46E+24	15.1%
1.64E+07	9.24E+12	7.70E+12	1.57E+12	2.46E+24	20.4%
1.65E+07	9.27E+12	7.04E+12	2.23E+12	4.96E+24	31.8%
1.65E+07	9.27E+12	7.50E+12	1.78E+12	3.18E+24	23.7%
1.64E+07	9.29E+12	7.41E+12	1.89E+12	3.88E+24	25.6%
Alpha		0.526	**Scaling Factor**		5.65E+04
Sum Errors Squared		2.61E+25	**Average Error**		12.0%

Table 4: Calculated Japanese Elasticity of Output with Respect to Capital (with α=0.526) using Sensitivity Analysis alongside Japanese GDP.

The data seems at first pretty good, other than the high but acceptable average percent error and an increasing percent error every year. However, if we look at our Solow model graphically, we notice an ever-widening gap. This is likely because of two main reasons. First (and more obvious) is their decreasing and inconsistent labor growth. Secondly, this may stem from a lack of spending from Japanese firms and weak business metabolism [4]. This once again shows that the Solow model is not entirely accurate at predicting growth.

5 Conclusion

We can conclude that under the right circumstances, the Solow model can be effectively applied to other countries; a great example of this is applying the Solow Model to the US. Germany does have room for improvement but otherwise also works. Further/already completed research into analyzing what factors are needed for an accurate Solow model prediction would help a lot with understanding economic growth.

However, the Solow model (or at least the application of Sensitivity Analysis) has many drawbacks. First, the Solow model fails to account for government, corruption, and institutional problems. For example, although India has a massive growing population, it lacks many advances in research, education, and infrastructure across the nation. Strict labor laws and corruption also hinder growth. In Japan's case, falling population numbers and a lack of spending from the private sector cause an increasing gap between our Solow model and actual numbers. Perhaps a better technology index could be used to better apply the Solow model.

The second drawback is the failure to capture the cyclical nature of economic growth. If we look back at the tables, notice how the errors cycle between positive and negative.

Although the Solow model does a good job of capturing long term economic growth, it fails to track this cyclical nature. In addition, our basic formulas to predict capital, labor, and technology also occasionally missed capturing cyclical nature (see figure 6). Further/completed research and better models can be developed on top of the Solow model to address these issues.

6 Further Implications

Firstly, it is important to note that Sensitivity Analysis is not the best way to apply the Solow model mathematically; there are likely better ways to apply the Solow model using our data.

Additionally, as mentioned earlier, a better technology index would help in applying the Solow Model. Since 1995, the dawn on the internet and increased computer/mobile phone usage have changed our world. Usage of AI may also be a key factor in measuring technological development in the 21st century.

Instead of looking at how accurate our Solow model is, it may actually be better to look analyze why the Solow model overpredicts growth. For India, although GDP is increasing, the Solow model shows that it might not be on the right track – if we apply this method to more countries, this may help us better check the growth and stability of developing countries.

References

[1] Caroline Banton. What is the neoclassical growth theory, and what does it predict?, November 2020. Accessed 21 November 2020.

[2] Paulo Barelli and Samuel de Abreu Pessôa. Inada conditions imply that production function must be asymptotically cobb–douglas. *Economics Letters*, 81(3):361–363, 2003.

[3] Jayant Krishna. Labor distortions could derail india's economic resurgence, June 2020.

[4] Yohei Matsuo and Daisuke Maruyama. Weak capital investment saps japan's economic growth, September 2022.

[5] Robert M. Solow. A contribution to the theory of economic growth. *The Quarterly Journal of Economics*, 70(1):65–94, 1956.

[6] Guntram B. Wolff and Alexander Roth. Understanding (the lack of) german public investment, June 2018.